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ON CALCULATION OF DYNAMIC ERROR PARAMETERS FOR THE RAWINSONDE AND RELATED SYSTEMS

By

Walter B. Miller

January 1972

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CONTENTS

INTRODUCTION

If a moving object is under observation by a given tracking system, the observations so made generally take the form of a sequence of vectors, each of which is related to a specific time, and whose components give estimates of the object's position in some convenient coordinate system. For a radar, this sequence is of the form $\{(r_i, a_i, e_i) | i=1,n\}$, where each vector in the sequence represents range, azimuth, and elevation, respectively, at times $\{t_i | i=1,n\}$ in the obvious fashion. In contrast, the rawinsonde system for balloon tracking gives rise to the family $\{(a'_i, e'_i, t'_i, p_i, h_i) | i=1,n\}$, where the components of a vector represent azimuth and elevation from the AN/GMD-( ) windset, and temperature, pressure and humidity from the radiosonde balloon. Here again all vectors are indexed by a time value. From each vector $(a'_i, e'_i, t'_i, p_i, h_i)$, a new vector (a'_i, e'_i, z_i) is derived through use of the hydrostatic equation. The family $\{(a'_i, e'_i, z_i) | i=1,n\}$ so formed represents position coordinates in a curvilinear system. It is this system which will be considered in the following study.

In the statistical model to be employed, a sequence of vectors arising in the fashion discussed previously will be considered as a single realization of an appropriate subprocess of a multivariate stochastic process. The index set for this process will be a set T containing $\{t_i | i=1,n\}$. For each t in T, the corresponding multivariate random variable will have as its mean a vector which represents the position of the object in an appropriate coordinate system and will have as its variance-covariance matrix a matrix Σ independent of t. The problems to be addressed in this study are concerned with obtaining estimates of the matrix Σ from a single realization of some multivariate subprocess. Such estimates will be designated as "dynamic", as they represent the behavior of the system in motion, as contrasted to static tests which obtain error estimates for a fixed configuration. The marginal processes are in general nonstationary, even in the weak sense [1] due to the variation of mean values with changing t, so that the problems involved may be considerable.

Chief among the problems encountered is the dependence of the estimates on trajectory. Suppose it is desired to obtain dynamic error parameters for a radar. Superficially, one might adopt the following procedure. The radar to be tested and a second radar of known error behavior are placed in close proximity and allowed to track an object simultaneously for a time interval T. The radars may be expected to give rise to vector sequences $\{(r_i, a_i, e_i) | i=1,n\}$ and $\{(r'_i, a'_i, e'_i) | i=1,n\}$, respectively, where the unprimed sequence will represent the radar of unknown error characteristics. As position and velocity are generally studied in a cartesian coordinate system, transform each to a single fixed reference system (X, Y, Z), obtaining sequences $\{(x_i, y_i, z_i) | i=1,n\}$

and $\{(x'_i, y'_i, z'_i)|i=1,n\}$. If calibration procedures are correct, up to errors introduced by the system itself, these values should be identical. Therefore, members of the vector sequence $\{(x_i-x'_i, y_i-y'_i, z_i-z'_i)|i=1,n\}$ should have near-zero components, and the variances and covariances required to determine the error behavior of the radar with reference to (x, y, z) position estimates may be calculated. With these values, the behavior of derivative estimates are determined. This procedure has been in common use in the past to estimate error behavior. To understand the difficulties introduced by such a procedure, additional background will be required and will be presented in the following section. This presentation will be made for two reasons. First, it is worthwhile from the standpoint of understanding error behavior introduced by mathematical transformations in a radar, and second it is necessary for the understanding of a technique to be introduced for determining dynamic error parameters for the rawinsonde.

ERROR BEHAVIOR IN THE TRANSFORMATION FROM SPHERICAL TO RECTANGULAR CARTESIAN COORDINATES

Let an object be moving along a trajectory $\{(\rho(t), \alpha(t), \varepsilon(t))|t\varepsilon T\}$ relative to a given spherical coordinate system. Suppose a radar is located at the origin of this coordinate system and oriented in such a fashion that a point $(\rho, \alpha, \varepsilon)$ in the system corresponds to range, azimuth, and elevation as seen by the radar. Let a sequence $\{(r_i, a_i, e_i)|i=1,n\}$ of observations be taken in the time interval T, along the given trajectory, these observations occurring at times $\{t_i|i=1,n\}$. It is desired to determine the trajectory of the moving object with respect to a rectangular coordinate system related to the first by the transformation

$$\begin{cases} x = \rho\cos\alpha\cos\varepsilon \\ y = \rho\sin\alpha\cos\varepsilon \\ z = \rho\sin\varepsilon \end{cases} \tag{1}$$

which will be called T.

From (1) the family $\{(r_i, a_i, e_i)|i=1,n\}$ gives rise to a new family $\{(x_i, y_i, z_i)|i=1,n\}$ in an obvious manner. If the first of these families is considered as a realization of a subprocess of a combined random process $\{(R_t, A_t, E_t)|t\varepsilon T\}$, $\{(x_i, y_i, z_i)|i=1,n\}$ becomes a realization of a subprocess of the joint stochastic process $\{(X_t, Y_t, Z_t)|t\varepsilon T\}$ defined for each t in T by

$$\begin{cases} X_t = R_t \cos A_t \cos E_t \\ Y_t = R_t \sin A_t \cos E_t \\ Z_t = R_t \sin E_t \end{cases}$$

Recall that for any t in T, (R_t, A_t, E_t) has mean vector $(\rho(t), \alpha(t), \varepsilon(t))$ and variance-covariance matrix Σ. Let R be a region in three-space in which T is at least of class C^1 (possesses continuous partial derivatives of order at least 1), and assume $\{(\rho(t), \alpha(t), \varepsilon(t)) | t\varepsilon T\}$ lies entirely in R. It is known [2] that for a given precision figure δ, there exist a transformation L and a constant λ dependent on δ such that

$$\Pr\{||L(R_t, A_t, E_t) - T(R_t, A_t, E_t)|| \geq \delta\} \leq \frac{tr(\Sigma)}{\lambda^2}, \tag{2}$$

where $tr(\Sigma)$ denotes the trace of Σ.

From (2), if $tr(\Sigma)$ is sufficiently small, the random vectors $L(R_t, A_t, E_t)$ and $T(R_t, A_t, E_t)$ differ only slightly in their behavior. The advantage of this lies in the fact that the variance-covariance matrix of $L(R_t, A_t, E_t)$ has the simple form $dT_t \Sigma d\tilde{T}_t$, where dT_t is the differential of the transformation T evaluated at $(\rho(t), \alpha(t), \varepsilon(t))$, and $d\tilde{T}_t$ denotes the transpose of dT_t [3]. The matrix dT_t has a simple form

$$dT_t = \begin{bmatrix} \cos\alpha(t)\cos\varepsilon(t) & -\rho(t)\sin\alpha(t)\cos\varepsilon(t) & -\rho(t)\cos\alpha(t)\sin\varepsilon(t) \\ \sin\alpha(t)\cos\varepsilon(t) & \rho(t)\cos\alpha(t)\cos\varepsilon(t) & -\rho(t)\sin\alpha(t)\sin\varepsilon(t) \\ \sin\varepsilon(t) & 0 & \rho(t)\cos\varepsilon(t) \end{bmatrix}. \tag{3}$$

From (3) the matrix $\Sigma'_t = dT_t \Sigma d\tilde{T}_t$ may easily be calculated.

As an example, let the object tracked be a balloon rising at a fixed rate, a, in still air. Assume the balloon is released a horizontal distance, b, along the line $\alpha = 0$. The parametric equations for the balloon trajectory are

$$\begin{cases} \rho(t) = (b^2 + a^2t^2)^{1/2} \\ \alpha(t) = 0 \\ \varepsilon(t) = \arccos\left(\dfrac{b}{[b^2 + a^2t^2]^{1/2}}\right). \end{cases}$$

The matrix dT_i becomes, for the time t,

$$\begin{bmatrix} \frac{b}{\sqrt{b^2 + a^2t^2}} & 0 & -at \\ 0 & b & 0 \\ \frac{at}{\sqrt{b^2 + a^2t^2}} & 0 & b \end{bmatrix}.$$

The matrix Σ'_t may now be calculated, obtaining

$$\begin{bmatrix} \frac{b^2\sigma_{11}}{b^2 + a^2t^2} + a^2t^2\sigma_{33} & 0 & \frac{bat\sigma_{11}}{b^2 + a^2t^2} - bat\sigma_{33} \\ 0 & b^2\sigma_{22} & 0 \\ \frac{bat\sigma_{11}}{b^2 + a^2t^2} - bat\sigma_{33} & 0 & \frac{a^2t^2\sigma_{11}}{b^2 + a^2t^2} + b^2\sigma_{33} \end{bmatrix}.$$

The matrix Σ'_t has, at $t=0$, the simple form

$$\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & b^2\sigma_{22} & 0 \\ 0 & 0 & b^2\sigma_{33} \end{bmatrix}.$$

As t increases, $\sigma'_{11}(t)$, the entry of Σ'_t in the first row and column, either decreases from σ^2_{11} to a minimum of

$$b\sqrt{\sigma_{33}}(2\sqrt{\sigma_{11}} - b\sqrt{\sigma_{33}})$$

at

$$t = \frac{b}{a}\sqrt{\left(\frac{\sigma_{11}}{b^2\sigma_{33}}\right)^{1/2} - 1},$$

then increases without bound thereafter, the case occurring when $\sigma_{11} > b\sigma_{33}$; or $\sigma'_{11}(t)$ increases without bound as t increases, the

case for $\sigma_{11} \leq b\sigma_{33}$. Clearly $\sigma'_{22}(t)$ remains constant for all values of t in T while σ'_{33} increases from $b^2\sigma^2_{33}$ at $t = 0$ approaching $c^2_{11} + b^2\sigma^2_{33}$ as t becomes large. The nonzero covariance term $\sigma'_{13}(t)=\sigma'_{31}(t)$ like $\sigma'_{11}(t)$, achieves an extremal value only if $\sigma_{11} > b\sigma_{33}$, in this case a maximum, then decreases without bound with increasing t values. In this case, when t is large, these terms resemble $-bat\sigma_{33}$. If $\sigma_{11}<b\sigma_{33}$, $\sigma'_{13}(t) = \sigma'_{31}(t)$ decreases without bound from $t = 0$, again resembling $-bat\sigma_{33}$ when t is large. It is not difficult to see the role played by b in the matrix Σ'_t. This emphasizes the dependence of the matrix Σ'_t on the trajectory chosen. Clearly, if two radars at different locations, say horizontal distances b_1 and b_2, observe the same balloon, then at a given time t, the associated variance-covariance matrices of the two random vectors produced would vary considerably. It would appear then that a realization of the process $\{(X_t, Y_t, Z_t)|t\varepsilon T\}$ is useless in the estimation of error parameters except in the grossest sense, so that other techniques must be utilized to describe the error behavior of the system. Such a technique will now be presented.

It is possible to approach the problem from a direct statistical standpoint, and results of some generality may be obtained. However, these techniques require knowledge or approximate knowledge of trajectory and, for the case of the rawinsonde, have the unfortunate characteristic of being unable to exclude induced balloon oscillations. For this case the results are included only as an appendix.

ESTIMATION OF DYNAMIC ERROR PARAMETERS FOR THE AN/GMD-( ) BY USE OF A RADAR

If a radar such as the T-9 or FPS-16 is available and in a proper configuration with the AN/GMD-( ), the following approach to the evaluation of error parameters of the AN/GMD-( ) shows promise. Let a radiosonde balloon be released and simultaneously tracked by the radar and the AN/GMD-( ). The output of the former of these devices consists of a sequence of vectors $\{(r_i, a_i, e_i)|i=1,n\}$ where the components of a vector (r_i, a_i, e_i) represent range, azimuth, and elevation estimates, respectively, for a time t_i in the interval $[t_1, t_n]$ under consideration. The latter system has as its output the vector sequence $\{(b_i, f_i)|i=1,n\}$. In this case, the components of a vector (b_i, f_i) are estimates of azimuth and elevation, respectively, at a time t_i identical to that time specified above. The vector sequences $\{(r_i, a_i, e_i)|i=1,n\}$ and $\{(b_i, f_i)|i=1,n\}$ will be considered as realizations of appropriate vector subprocesses of the continuous vector processes $\{(R_t, A_t, E_t)|t\varepsilon T\}$ and $\{(B_t, F_t)|t\varepsilon T\}$, respectively, where $[t_1, t_n]$ is contained in T. The mean for the former of these processes will be the vector function $\{(\rho(t), \alpha(t), \varepsilon(t))|t\varepsilon T\}$ which gives the true trajectory of the balloon in the obvious spherical coordinate system associated with the radar. The vector function $\{(\beta(t), \phi(t))|t\varepsilon T\}$ plays an exactly similar role

with the GMD. It will also be assumed that the former sequence has variance-covariance matrix Σ independent of t in T, and the latter the variance-covariance matrix Γ, likewise independent of t in T. Now let (λ, δ, η) be a vector giving in component order range, azimuth and elevation of the radar under consideration with respect to the GMD. In other words, (λ, δ, η) are the spherical coordinates of the radar with respect to the obvious system located at the GMD. (See Figure 1.) It will be assumed throughout that the radar and the GMD are in parallel orientation. It is clear that if λ is sufficiently large, this would not in general be the case, this due to the curvature of the earth. The equations relating $(\beta(t), \phi(t))$ and $(\rho(t), \alpha(t), \varepsilon(t))$ will be more complicated when the earth's curvature is included than when it is not; therefore, from a standpoint of simplicity, the parallel orientation model is desirable. Also, as will be made clear later, if λ is chosen too large, the method for evaluating dynamic parameters to be presented now will fail. Then concern will be only with λ so small that curvature of the earth presents a negligible effect.

Under these circumstances, and under suitable restrictions, the following equations may be shown to hold:

$$\beta(t) = \arctan\left[\frac{\rho(t)\cos\varepsilon(t)\sin\alpha(t) + \lambda\cos\eta\sin\delta}{\rho(t)\cos\varepsilon(t)\cos\alpha(t) + \lambda\cos\eta\cos\delta}\right]$$

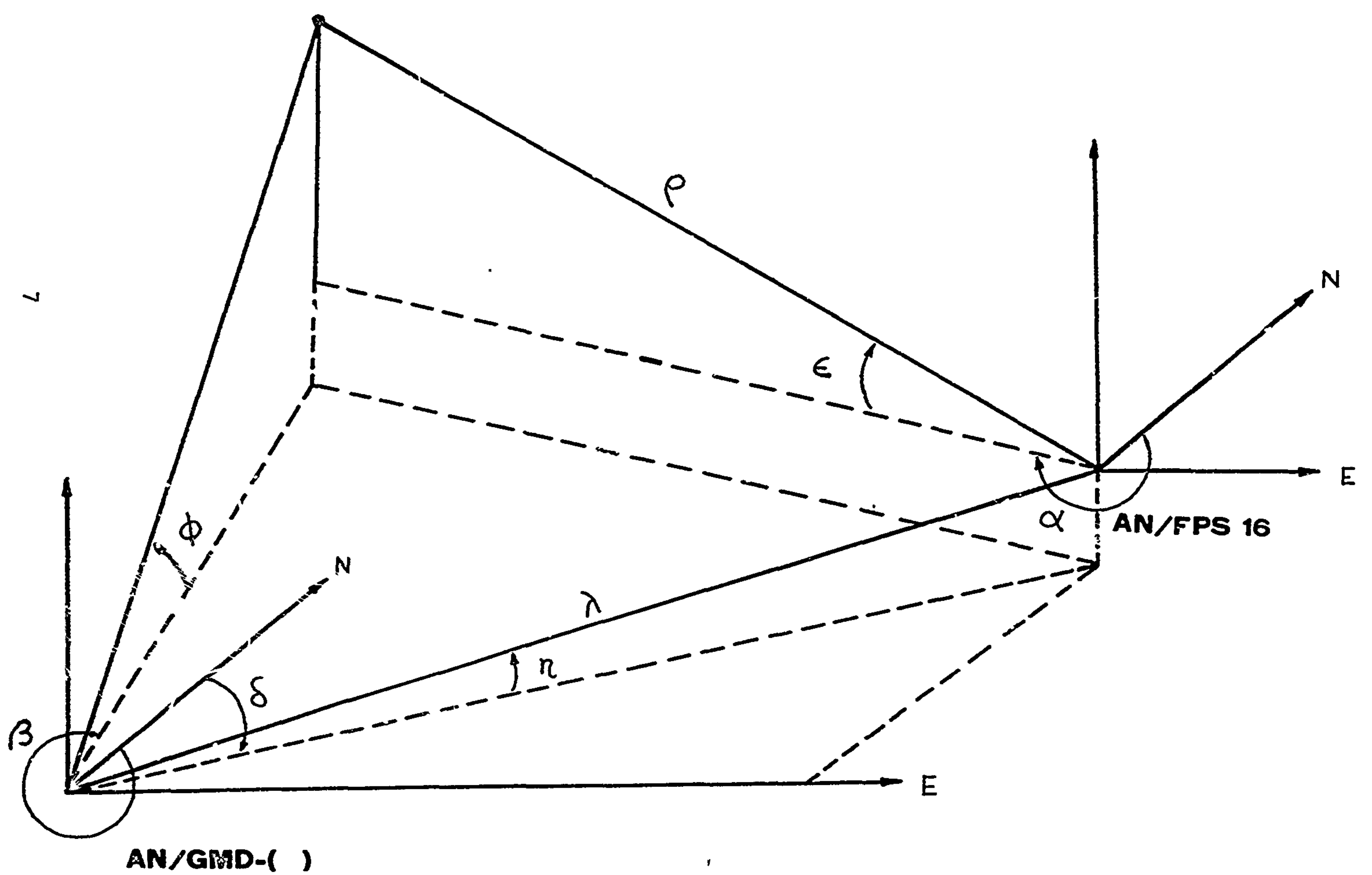
$$\phi(t) = \arctan\left[\frac{\rho(t)\sin\varepsilon(t) + \lambda\sin\eta}{\sqrt{\psi(t)}}\right] \tag{4}$$

where $\psi(t)$ is given by

$$\psi(t) = [(\rho(t)\cos\varepsilon(t))^2 + (\lambda\cos\eta)^2 + 2\lambda\rho(t)\cos\eta\cos\varepsilon(t)\cos(\alpha(t)-\delta)].$$

The equations in (4) define a transformation of Euclidian 3-space into Euclidian 2-space which will be denoted by T^*. It follows by definition that $\{T^*(\rho(t), \alpha(t), \varepsilon(t))|t\varepsilon T\}$ is identically the vector function $\{(\beta(t), \phi(t))|t\varepsilon T\}$ if quadrant ambiguities are considered. The vector process $\{T^*(R_t, A_t, E_t)|t\varepsilon T\}$ would be expected to exhibit behavior in its mean similar to $\{(B_t, F_t)|t\varepsilon T\}$. The complicated nature of T^* prohibits direct calculation of the mean and variance-covariance matrices of the various vectors in $\{T^*(R_t, A_t, E_t)|t\varepsilon T\}$ even under assumptions of joint normality of the process $\{(R_t, A_t, E_t)|t\varepsilon T\}$. In this case it becomes necessary to resort to approximation. The magnitude of the elements of Σ suggests that satisfactory approximations of both the mean and the variance-covariance matrix of $\{T^*(R_t, A_t, E_t)|t\varepsilon T\}$ are possible in terms of Σ and $\{(\rho(t), \alpha(t), \varepsilon(t))|t\varepsilon T\}$ [2]. In this light, the approximation to the desired variance-covariance matrix of $T^*(R_t, A_t, E_t)$ is simply $dT^*_t\Sigma dT^{*'}_t$ where dT^*_t is the differential of the transformation

FIGURE 1. RELATED MEASUREMENTS OF (β,ϕ) FOR THE AN/GMD-( ) AND (ρ,α,ϵ) FOR THE AN/FPS 16 RADAR.



T^* [3] a^+ $(\rho(t), \alpha(t), \varepsilon(t))$, and dT^*_t denotes the transpose of this matrix. The calculation of dT^*_t is routine but tedious, and the following definitions will be made to simplify notation.

$$\Lambda(t) = (\rho(t)\cos\varepsilon(t))^2 + (\lambda\cos\eta)^2 + 2\lambda\cos\eta\rho(t)\cos\varepsilon(t)\cos(\alpha(t)-\delta).$$

For $i=1, 2$, $j=1, 2, 3$, let $\mu^*(i,j,t)$ designate the i, jth entry of dT^*_t. Then

$$\mu^*(1,1,t) = \frac{\lambda\cos\eta\cos\varepsilon(t)\sin(\alpha(t)-\delta)}{\psi(t)}$$

$$\mu^*(1,2,t) = \frac{[\rho(t)\cos\varepsilon(t)]^2 + \rho(t)\lambda\cos\eta\cos\varepsilon(t)\cos(\alpha(t)-\delta)}{\psi(t)}$$

$$\mu^*(1,3,t) = \frac{\lambda\cos\eta\sin\varepsilon(t)\sin(\alpha(t)-\delta)}{\psi(t)}$$

$$\mu^*(2,1,t) = \frac{\lambda^2\cos\eta[\sin\varepsilon(t)\cos\eta - \cos\varepsilon(t)\sin\eta\cos(\alpha(t)-\delta)]}{\Lambda\sqrt{\psi(t)}}$$

$$- \frac{\lambda\rho(t)\cos\varepsilon(t)[\sin\eta\cos\varepsilon(t) - \cos\eta\sin\varepsilon(t)\cos(\alpha(t)-\delta)]}{\Lambda\sqrt{\psi(t)}}$$

$$\mu^*(2,2,t) = \frac{[\rho(t)\sin\varepsilon(t) + \lambda\sin\eta][\lambda\rho(t)\cos\eta\cos\varepsilon(t)\sin(\alpha(t)-\delta)]}{\Lambda\sqrt{\psi(t)}}$$

$$\mu^*(2,3,t) = \frac{\rho^3(t)\cos\varepsilon(t) + 2\lambda\rho^2(t)\cos^2\varepsilon(t)\cos\eta\cos(\alpha(t)-\delta)}{\Lambda\sqrt{\psi(t)}}$$

$$+ \frac{\lambda\rho(t)[\rho(t)\sin\varepsilon(t) + \lambda\sin\eta][\sin\eta\cos\varepsilon(t) + \cos\varepsilon\sin\varepsilon(t)\cos(\alpha(t)-\delta)]}{\Lambda\sqrt{\psi(t)}}$$

Observe that the following relations hold.

$$\lim_{\rho(t)\to\infty}\mu^*(1,2,t) = \lim_{\rho(t)\to\infty}\mu^*(2,3,t) = 1$$

$$\lim_{\rho(t)\to\infty}\mu^*(1,j,t) = \lim_{\rho(t)\to\infty}\mu^*(2,k,t) = 0 \quad j\neq 2,\ k\neq 3.$$

Hence in the case of very large $\rho(t)$, the matrix dT^*_t resembles the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $dT^*_t \Sigma d\tilde{T}^*_t$ resembles

$$\begin{bmatrix} \sigma_{22} & 0 \\ 0 & \sigma_{33} \end{bmatrix},$$

the lower principal submatrix of Σ. It follows that for $\rho(t)$ sufficiently large, the bivariate random variable $([B_t - T^*_1(R_t, A_t, E_t)], [F_t - T^*_2(R_t, A_t, E_t)]$ has as its variance-covariance matrix a matrix resembling

$$\begin{bmatrix} \sigma_{22}+\gamma_{11} & \sigma_{23}+\gamma_{12} \\ \sigma_{32}+\gamma_{21} & \sigma_{33}+\gamma_{22} \end{bmatrix},$$

where the dependence of the matrix on the trajectory $\{(\rho(t), \alpha(t), \varepsilon(t)) | t\varepsilon T\}$ has been nearly eliminated. Clearly the variance-covariance matrix associated with any other time t' will also have the above form if $\rho(t') \geq \rho(t)$. It should be noted that the estimate of Γ will be $dT^*_t \Sigma dd\tilde{T}^*_t - \Sigma'$, where Σ' is the lower principal 2x2 submatrix of Σ. As $\rho(t)$ increases, the matrix $dT^*_t \Sigma d\tilde{T}^*_t - \Sigma'$ becomes increasingly less dependent on the trajectory and gives an increasingly more valid representation of Γ. In addition, an examination of $dT^*_t \Sigma d\tilde{T}^*_t$ yields that terms of the nature of

$$\frac{\lambda}{\rho(t)}$$

will be present. Clearly then if it is desired that these terms go to zero, $\rho(t)$ must be large with respect to λ. This may be facilitated if a configuration can be chosen such that λ is small with respect to ranges which may be encountered. If such a configuration is not possible, practical attempts at estimating Γ in this fashion will fail, and in general will result in overestimation of the magnitudes involved. For this reason, it is clear that the magnitude of λ required for successful evaluation of dynamic parameters will be so small that curvature problems are of no consequence. It should be mentioned at this time that the vectors of means of the members of the family $\{T^*(R_t, A_t, E_t) | t\varepsilon T\}$ are biased away from the respective vectors in $\{T^*(\rho(t), \alpha(t), \varepsilon(t)) | t\varepsilon T\}$. These biases, however, may be shown to be very small when $\rho(t)$ is large. The demonstration of this fact is routine but tedious and will be

omitted. It follows that if $\rho(t)$ is large, the bivariate process $\{([B_t - \psi_1(R_t, A_t, E_t)], [F_t - \psi_2(R_t, A_t, E_t)]) | t \epsilon T\}$ will have mean near zero and independent of t in T. In this portion of the trajectory the process is very near to weakly stationary and hence behaves as a sample. One may then estimate the diagonal members of the variance-covariance matrix of this process and hence obtain an estimate of Γ.

In Table I, estimates of Γ are derived using FPS-16 radars and AN/GMD-IB windsets at various base lines, or λ values. The scarcity of data is recognized, but the tendencies indicated bear out the theoretical development, and would seem to indicate that the AN/GMD-IB is operating at near to engineering specifications insofar as measurement of azimuth and elevation angles is concerned.

The problems of greatest magnitude associated with the rawinsonde system appear to be related to errors in height. A systematic study of the effects of height errors on position estimates for the rawinsonde system is found, for example, in [4] and indicates that for small elevation angles, system performance depends critically on height errors, with small excursions in height causing large excursions in corresponding position estimates.

Errors in height may be divided into two classes. The first are errors due to systematic biases and random fluctuations in the instruments measuring temperature, pressure, and humidity. The second are errors due to physical situations which may cause variations in the validity of the hydrostatic equation. It is clear that errors such as these can vary with height and from run to run. If an individual trajectory is examined, only the excursions due to random fluctuations in the instrumentation of the radiosonde package are apparent. The remaining errors generally take the form of biases, the presence of which is not obvious from the examination of a single trajectory unless this trajectory is compared with a trajectory measured by a device whose error behavior results in a trajectory which is not so greatly biased away from the absolute; a trajectory such as that measured by an FPS-16. The exact error behavior of height measurements due to random fluctuations in temperature, pressure, and humidity is difficult to obtain, even approximations of this quantity. This is due to the complexity of the mathematical operations involved and the number of interacting factors. However, the portion of the variance of height estimators due to this aspect is believed by the author to be quite small in relation to the remaining contributors. This explains the smoothness and small excursions observed in a single trajectory. It is only when many trajectories are available which may be compared with absolute trajectories that the effects of the remaining contributors become apparent. It is suspected by the author that the effects of the latter contributors may be an order of magnitude greater than the former. As has been remarked, the magnitudes

TABLE I

RESULTS OF DYNAMIC PARAMETER ESTIMATION FOR TWO AN/GMD-1B WINDSETS, UTILIZING THREE FPS-16 RADARS AT VARIOUS SLANT RANGES.

AN/GMD-1B Employed	FPS-16 Radar Employed	Slant Range From AN/GMD-1B To Radar	Dynamic Estimate of Standard Deviation of Elevation Angle	Dynamic Estimate of Standard Deviation of Azimuth Angle
Small Missile Range	113	14.6 km	2.18 deg.	2.59 deg.
Small Missile Range	113	14.6 km	.94 deg.	1.63 deg.
WSD	113	4.7 km	.28 deg.	.11 deg.
WSD	113	4.7 km	.25 deg.	.27 deg.
WSD	114	4.7 km	.11 deg.	.15 deg.
WSD	114	4.7 km	.12 deg.	.30 deg.
Mobile Unit	112	86 meters	.06 deg.	.06 deg.
Mobile Unit	112	86 meters	.15 deg.	.13 deg.
Mobile Unit	113	38 meters	.03 deg.	.08 deg.
Mobile Unit	[illegible]	38 meters	.05 deg.	.07 deg.

of the errors in height may be expected to vary with height, and from run to run; a single error parameter, or for that matter any number of error parameters, may not suffice to describe the system adequately. In this case one is forced either to calculate bounds, if possible, or to seek situations for which error parameters are stable. Toward this end the following procedure may be useful for determining average absolute height errors for the rawinsonde system.

Let $\theta(t)$ be defined for each t in T by

$$\theta(t) = \rho(t)\sin\varepsilon(t) + \lambda\sin\eta.$$

Here $\rho(t)$, $\varepsilon(t)$, λ, η have the meanings ascribed previously. It is not difficult to see that $\theta(t)$ is the height of the balloon under observation at time t in T as seen from the AN/GMD-( ). A reasonable estimator of this quantity based on observations from the radar is

$$\theta_t = R_t \sin E_t + \lambda\sin\eta.$$

For any t in T, by the same logic as presented earlier, the variance of θ_t is approximated by

$$\mathrm{Var}(\theta_t) \approx \sin^2\varepsilon(t)\sigma_{11} + \rho^2(t)\cos^2\varepsilon(t)\sigma_{33}.$$

Now suppose that

$$\rho(t) \leq \sqrt{\frac{\sigma_{11}}{\sigma_{33}}}\,.$$

Then

$$\rho^2(t) \leq \frac{\sigma_{11}}{\sigma_{33}}$$

or

$$\rho^2(t)\sigma_{33} \leq \sigma_{11}.$$

If it is assumed that $0 < \varepsilon(t) \leq \pi/2$, $\cos\varepsilon(t) > 0$ and $\rho^2(t)\cos^2\varepsilon(t)\sigma_{33} \leq \cos^2\varepsilon(t)\sigma_{11}$; hence, $\rho^2(t)\cos^2\varepsilon(t)\sigma_{33} \leq (1-\sin^2\varepsilon(t))\sigma_{11}$ and $\rho^2(t)\cos^2\varepsilon(t)\sigma_{33} + \sin^2\varepsilon(t)\sigma_{11} \leq \sigma_{11}$. From this it follows that $\mathrm{Var}\theta_t \leq \sigma_{11}$ when

$$\rho(t) \leq \frac{\sigma_{11}}{\sigma_{33}}\,.$$

(Accepted values [5] for the error behavior of the FPS-16 give $\sigma_{11} = 15$ feet, $\sigma_{33} = .01$ degree; hence, the error in θ_t will be bounded by 15 feet for slant ranges up to 83,000 feet.)

If Z_t represents the estimate of balloon height from the rawinsonde system, it follows immediately that $\mathrm{Var}(Z_t) \geq \mathrm{Var}(Z_t-\theta_t) - \sigma_{11}$ as long as

$$\rho(t) \leq \frac{\sigma_{11}}{\sigma_{33}} .$$

Observing that one may choose a portion of a given trajectory to satisfy this condition, and also that for any t in the given range the expected value of $(Z_t-\theta_t)$ should be zero, an estimate of an average variance of $(Z_t-\theta_t)$ may be calculated for this interval and hence a lower bound for $\mathrm{Var}(Z_t)$ in that range. It is again to be stressed that such estimates will vary from run to run and could only be expected to yield gross behavior characteristics. However, in many circumstances gross behavior characteristics are better than wrong behavior characteristics or none at all.

APPENDIX

A General Statistical Approach

Let $X = (X_1, X_2, \ldots, X_n)$ be a multivariate random variable with mean $x = (x_1, x_2, \ldots, x_n)$ and variance-covariance matrix Σ. Let T be a transformation on Euclidian n-space of class C^1 in a region R containing x. Suppose that T has parametric representation

$$
\begin{aligned}
y_1 &= f_1(x_1, x_2, \ldots, x_n) \\
y_2 &= f_2(x_1 \ldots x_n) \\
\ldots & \quad \ldots \quad \ldots \quad \ldots \\
y_n &= f_n(x_1, x_2, \ldots, x_n)
\end{aligned}
\tag{1}
$$

Consider the multivariate random variable $Y = (Y_1, Y_2, \ldots, Y_n)$ obtained from $(X_1, X_2, \ldots, X_n)$ and T in the obvious fashion. From the multivariate random variable $Y = T(X)$ it is desired to recover information about X, in particular the vector of means and variance-covariance matrix. The following theorem will be required which is similar to Theorem 25 of chapter 5 of [3], and is included for reference.

Theorem 1: Let T be a transformation on Euclidian n-space of class C^1 in a region R containing x. Let dT represent the differential [3] of T at x, and assume dT is nonsingular or equivalently that the Jacobian of T does not vanish at x. If $y - T(x)$, then there exists a neighborhood J about x which is mapped in a 1-1 fashion onto a neighborhood $T(J)$ of y, and on $T(J)$, a transformation T^{-1} may be defined class C^1 in $T(J)$ such that for any z in J, $T^{-1}(T(z)) = Z$, and moreover, if dT^{-1} is the differential of T^{-1} evaluated at y, $dTdT^{-1} = I$, when I is the identity matrix.

The following theorem may now be established.

Theorem 2: Let X, T be as previously defined. Let T^{-1} be as in theorem 1. If $||v||$ is defined for e n-vector v by

$$||v|| = \sqrt{\sum_{j=1}^{n} v_j^2} \, ,$$

then as T^{-1} is a measurable transformation there exists a number $\eta>0$ such that

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$$\Pr\{X \neq T^{-1}(Y)\} \leq \frac{tr(\Sigma)}{\eta^2} .$$

Here Pr denotes probability, $tr(\Sigma)$ the trace of Σ.

Proof: Choose η so small that $\{z \mid \|z-x\|<\eta\}$ is contained in J. This assures that T^{-1} is defined on $T(\{z \mid \|z-x\|<\eta\})$, hence $\{z \mid \|z-x\|<\eta\}$ is contained in $\{z \mid z=T^{-1}(T(z))\}$. Now consider the event $\{X \neq T^{-1}(Y)\}$. (Observe that T^{-1} will be measurable.) It is clear that $\{X \neq T^{-1}(Y)\} \subset \{\|X-x\| \geq \eta\}$ so that $\Pr\{X \neq T^{-1}(Y)\} \leq \Pr\{\|X-x\| \geq \eta\}$. However,

$$\Pr\{\|X-x\| \geq \eta\} = \Pr\{\sum_{j=1}^{n} (X-x)^2 \geq \eta^2\}.$$

From this it follows (see [6], section 15.7) that

$$\Pr\{\|X-x\| \geq \eta\} \leq \frac{\sum_{j=1}^{n} \sigma_{jj}}{\eta^2} = \frac{tr(\Sigma)}{\eta^2} .$$

This proves the theorem.

This theorem states that the probability that X and $T^{-1}(Y)$ will differ depends on the size of the trace of Σ, and on the area of definition of T^{-1}. If the nature of T is such that η is very small, unless $tr(\Sigma)$ is correspondingly small, it may occur that in a probability sense Y may furnish little information about X.

Estimation of Parameters

To estimate the matrix Σ from a sample of the multivariate random variable Y, one may choose to estimate the variance-covariance matrix of Y, call it Λ, and then take $dT^{-1}\Lambda d\tilde{T}^{-1}$ as the desired estimate. This estimate will be biased, especially since the expected value of Y is not necessarily $T(X)$. However, if $tr(\Sigma)$ is sufficiently small, this will not be significant. In dealing with multivariate stochastic processes, affairs become more complicated. Let $\{X_t \mid t \in T\}$ be such a process. Let the vector of means of X_t be given for any time t in T by the vector function $X(t)$, and let Σ be the variance-covariance matrix, independent of t. Let T be a measurable transformation on Euclidian n-space of class C^1 in a region R containing $\{X(t) \mid t \in T\}$. For each t in T, let $Y_t = T(X_t)$. The multivariate process $\{Y_t \mid t \in T\}$ so defined will no longer have variance-covariance matrices independent of T. Let Λ_t denote the variance-covariance matrix

of Y_t for each t in T. Observe that no nontrivial estimate of Λ_t is possible from a single realization of $\{Y_t|t\varepsilon T\}$, hence a technique such as described above for finding Σ is not possible.

It has been shown [2], however, that for each t in T there exists a transformation L_t on n-space such that $L_t(X_t)$ is a multivariate random variable with mean $T(X(t))$ and variance-covariance matrix $dT_t\Sigma d\tilde{T}_t$, where dT_t represents the differential of T at $x(t)$. Moreover, for any chosen precision figure ε, there exists a number $\delta(\varepsilon)$ such that

$$\Pr\{||Y_t - L_t(X_t)|| \geq \varepsilon\} \leq \frac{tr(\Sigma)}{\delta(\varepsilon)} .$$

If for a chosen $\varepsilon>0$, $tr(\Sigma)$ is sufficiently small that $\{L_t(X_t)|t\varepsilon T\}$ is a uniformly satisfactory substitute for $\{Y_t|t\varepsilon T\}$, the following approach is possible. For i and j among 1, 2, ..., n, let $\mu(i,j,t)$ and $\sigma'(i,j,t)$ represent the i, jth entries of dT_t and $dT_t\Sigma d\tilde{T}_t$, respectively. Now assume Σ is diagonal so that one may always write

$$\sigma'(i,j,t) = \sum_{k=1}^{n} \mu(i,k,t)\mu(j,k,t)\sigma_{kk}. \tag{1}$$

Since $dT_t\Sigma d\tilde{T}_t$ is symmetric, it is possible to find,

$$\frac{n(n+1)}{2}$$

distinct equations of the above form of which at most n may be independent. A reasonable choice of n equations is obtained by setting i = j, obtaining for $\ell = 1, 2, \ldots, n$,

$$\sigma'(\ell,\ell) = \sum_{k=1}^{n}\mu^2(\ell,k,t)\sigma_{kk}.$$

Observe that the fact that dT_t is nonsingular does not imply that this system is nonsingular. Now suppose that for $[t_1,t_2]$ an interval in T, $\sigma(i,j,t)$ and $\mu(i,j,t)$ are integrable functions of t. One may then write from (1)

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\sigma'(i,j,t)dt = \sum_{k=1}^{n}\left[\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\mu(i,k,t)\mu(j,k,t)dt\right]\sigma_{kk}. \tag{2}$$

Like (1), (2) is also a linear equation in the σ_{kk} with the coefficients completely determined by the knowledge of $\{x(t)|t\varepsilon T\}$ and τ. If the integral

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\sigma'(i,j,t)dt$$

is known, for some n distinct values of (i,j), and the resulting system is nonsingular, the σ_{kk} are completely determined. However, these values are not known. If estimates of some satisfactory set of these integrals exist, however, this may lead to estimates of the σ_{kk}.

Consider the system defined for $\ell = 1, 2, \ldots, n$ by

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\sigma'(j,j,t)dt = \sum_{k=1}^{n}\left[\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\mu^2(j,j,t)dt\right]\sigma_{kk}.$$

Let $[t_1,t_2]$ be divided into m-1 equal intervals of length Δ by $t_1 = s_1 < s_2 < \ldots < s_m = t_2$. Let $X_j = Xs_j$ for $j=1, 2, \ldots, m$ and let x_j be the mean of X_{ij}. Define $\{L_i(X_j)|j=1,m\}$ in the obvious manner, and let $L_{j,i}$ be the ith component of L_j. Now for $i = 1, 2, \ldots, n$, define

$$S_i = \frac{1}{m}\sum_{j=1}^{m}[L_{j,i}(X_j) - \tau_i(x_j)]^2.$$

It follows that

$$E(S_i) = \frac{1}{m}\sum_{j=1}^{m}E[L_{j,i}(X_j) - \tau_i(X_j)]^2 = \frac{1}{m}\sum_{j=1}^{m}\sigma'(i,i,j)$$

$$= \frac{1}{t_2-t_1}\sum_{j=1}^{m}\sigma'(i,i,s_j)\Delta.$$

Since $\sigma'(i,i,t)$ is integrable on $[t_1,t_2]$, as Δ approaches zero,

$$\frac{1}{t_2-t_1}\sum_{j=1}^{m}\sigma'(i,i,s_j)\Delta,$$

approaches

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\sigma^2(i,i,t)\,dt.$$

The linear system

$$S_j = \sum_{k=1}^{n}\left[\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\mu^2(j,k,t)\,dt\right]\omega_k \qquad j=1,\ 2,\ \ldots,\ n$$

may now be solved for ω_k, providing that the system is nonsingular. If it should occur that the system is singular, other members of the

$$\frac{n(m+1)}{2}$$

distinct equations of this form may be tried or $[t_1,t_2]$ varied to obtain a sufficient number of independent equations.

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